Rolling Element Bearing Stiffness Matrix Determination

Gearbox Reliability Collaborative Meeting 2012

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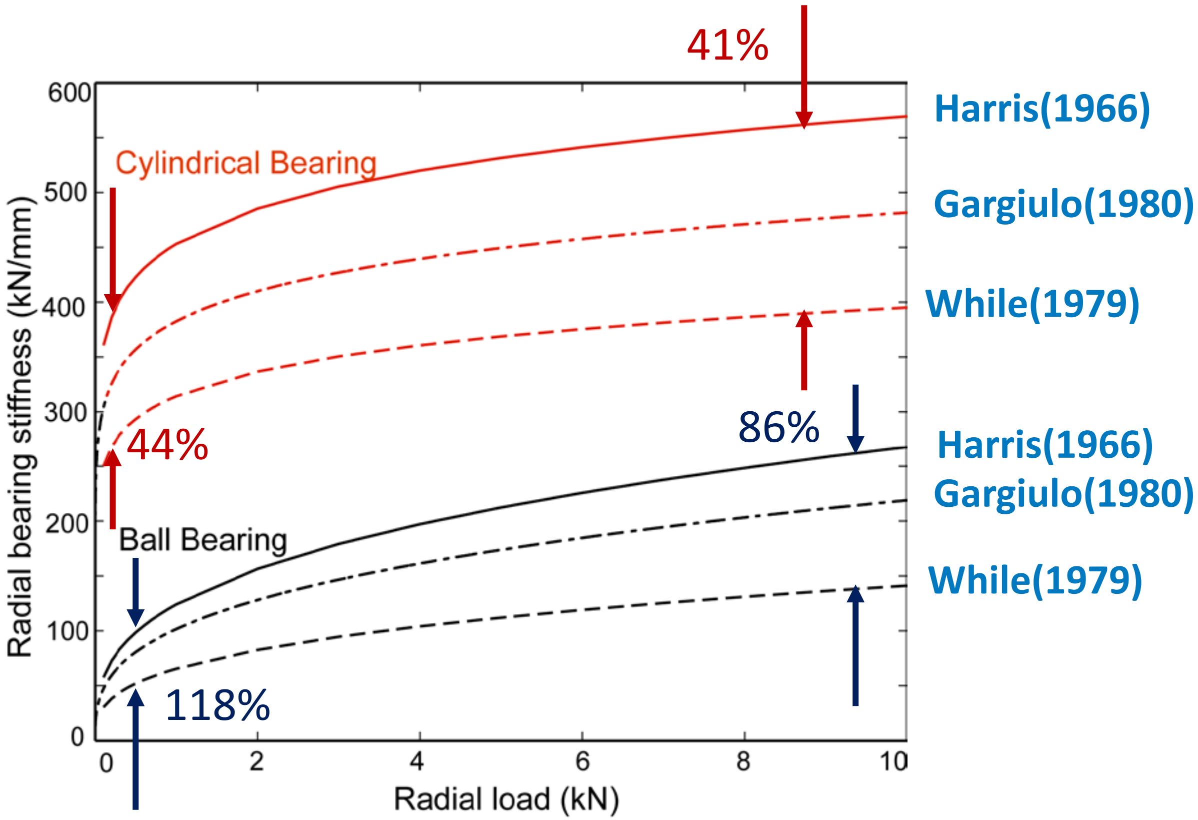
Motivation

- **Current theoretical models differ in stiffness estimates**
- **Uncertainty in stiffness estimate from manufactures**



Motivation

- **Limited work on stiffness matrix in the literature**
  - Diagonal matrix approximation typically used
- **Elastic deformation of race causes off-diagonal terms**

$$\begin{bmatrix} k_{xx} & k_{xy} & k_{xz} & k_{x\theta_x} & k_{x\theta_y} & 0 \\ & k_{yy} & k_{yz} & k_{y\theta_x} & k_{y\theta_y} & 0 \\ & & k_{zz} & k_{z\theta_x} & k_{z\theta_y} & 0 \\ & & & k_{\theta_x\theta_x} & k_{\theta_x\theta_y} & 0 \\ & \mathit{Symmetric} & & & k_{\theta_y\theta_y} & 0 \\ & & & & & 0 \end{bmatrix}$$

Coupling between the radial and rotational displacement

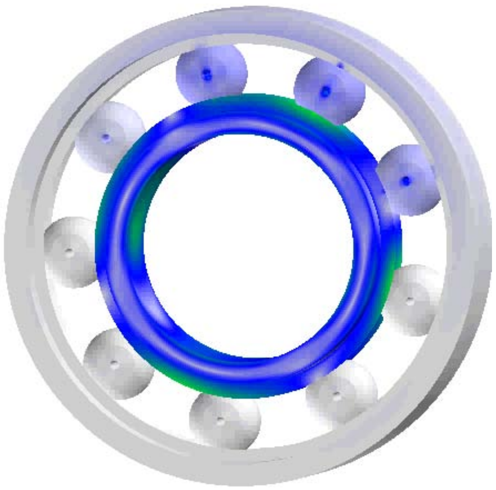
Coupling between the axial and rotational displacement

Coupling between the axial and radial displacement

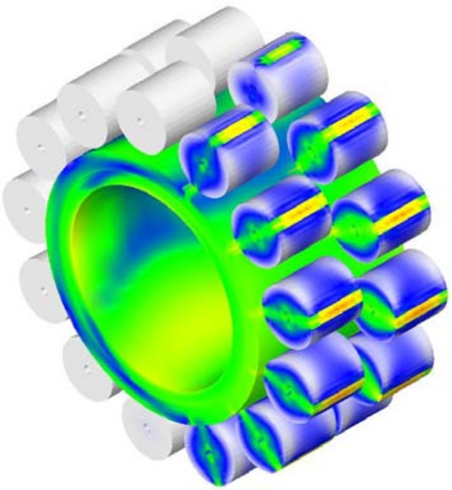
Finite Element/Contact Mechanics Model

- **3D finite element model includes micro-geometry**
- **Analyze contact between rolling elements and races**
  - Contact searched at every time instant as bearing rotates

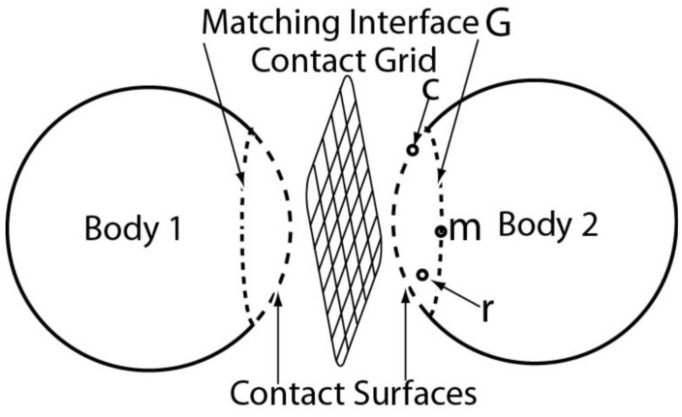
Radial Ball Bearing



Cylindrical Bearing



Contact Grid



Bearing Stiffness Computation Method

- **Bearing contact can be modeled by stiffness matrix**
  - Stiffness matrix changes with applied loads/moments

$$X = [x, y, z, \theta_x, \theta_y, \theta_z]$$

$$F = [f_x, f_y, f_z, M_x, M_y, M_z]$$

$$K = \begin{bmatrix} \frac{\partial f_x}{\partial x} & & \frac{\partial f_x}{\partial \theta_z} \\ & \ddots & \\ \frac{\partial M_z}{\partial x} & & \frac{\partial M_z}{\partial \theta_z} \end{bmatrix}$$

- **Numerical Jacobian used to compute K**

$$K = \frac{\partial F}{\partial X} \approx \begin{cases} \dfrac{[F(X+\delta) - F(X)]}{\delta} \\ \dfrac{[F(X+\delta) - F(X-\delta)]}{2\delta} \\ \dfrac{[8F(X+\frac{\delta}{2}) - 8F(X-\frac{\delta}{2}) - F(X+\delta) + F(X-\delta)]}{6\delta} \\ \vdots \end{cases}$$

Accuracy Order of Finite Element Analysis

- **Order of Jacobian approximation formula should be comparable to the accuracy order of FEA**
- **Method to obtain the accuracy order of FEA**

$$V_h = V + \underbrace{c_1 h^{p_1} + c_2 h^{p_2} + \ldots}_{e_h}$$

$$\frac{e_{h_3} - e_{h_2}}{e_{h_2} - e_{h_1}} = \frac{V_{h_3} - V_{h_2}}{V_{h_2} - V_{h_1}} \approx \frac{h_2^{p_1} - h_3^{p_1}}{h_1^{p_1} - h_2^{p_1}}$$

$$p_1 = \frac{log\{(\frac{V_{h_3} - V_{h_1}}{V_{h_3} - V_{h_2}})\frac{log\frac{h_2}{h_3}}{log\frac{h1}{h3}}\}}{log(\frac{h_1}{h_2})}$$

V_h : finite element solution
V : exact solution
h : finite element size
p_1 : order of accuracy

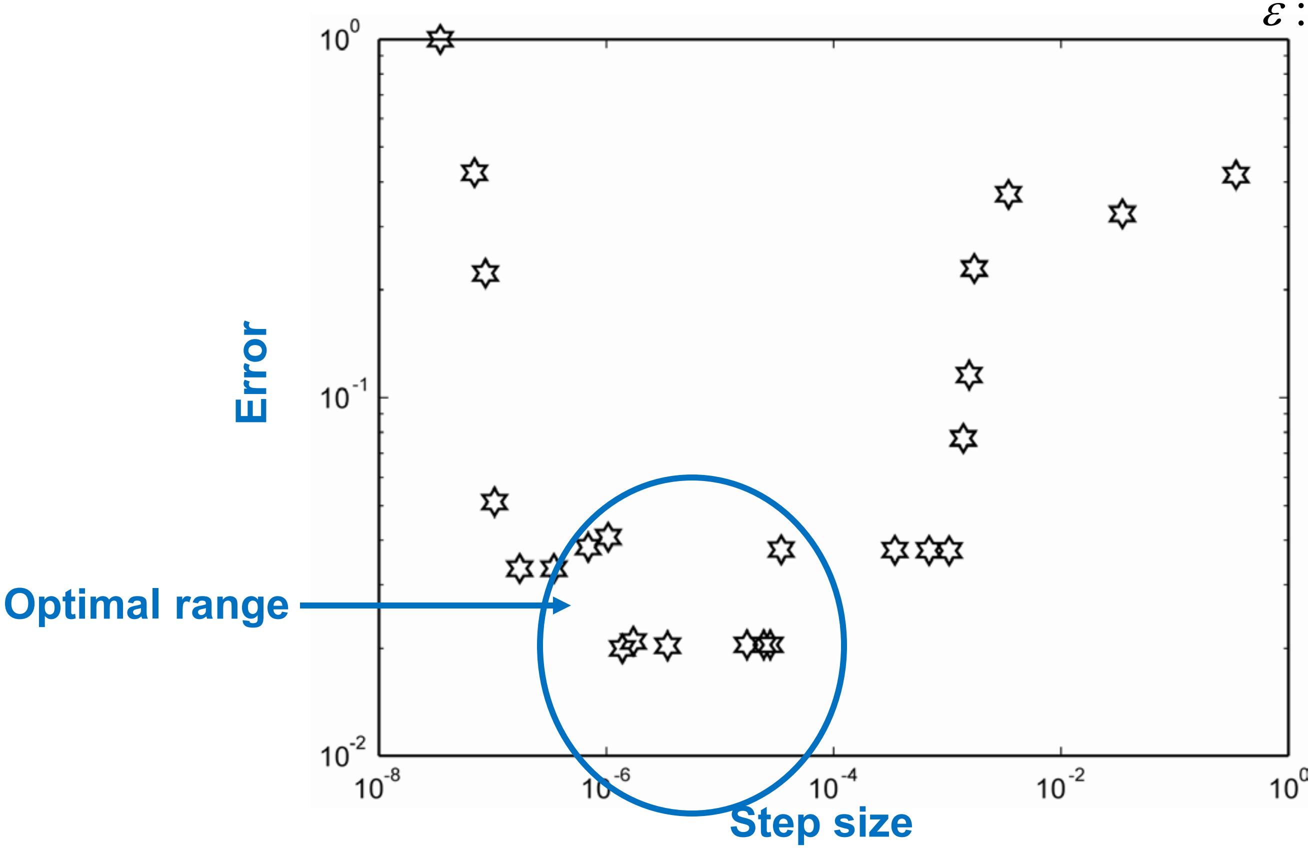
Accuracy Order	FEA	FEA/Contact
p1	1.11	1.94

Step Size Selection

- **Step size selection is essential for stiffness accuracy**
  - Round off error comes into play with extremely small step size
- **Analytical prediction of the step size**

$$\delta = \varepsilon^{\frac{1}{3}} \approx 5\times 10^{-6}$$

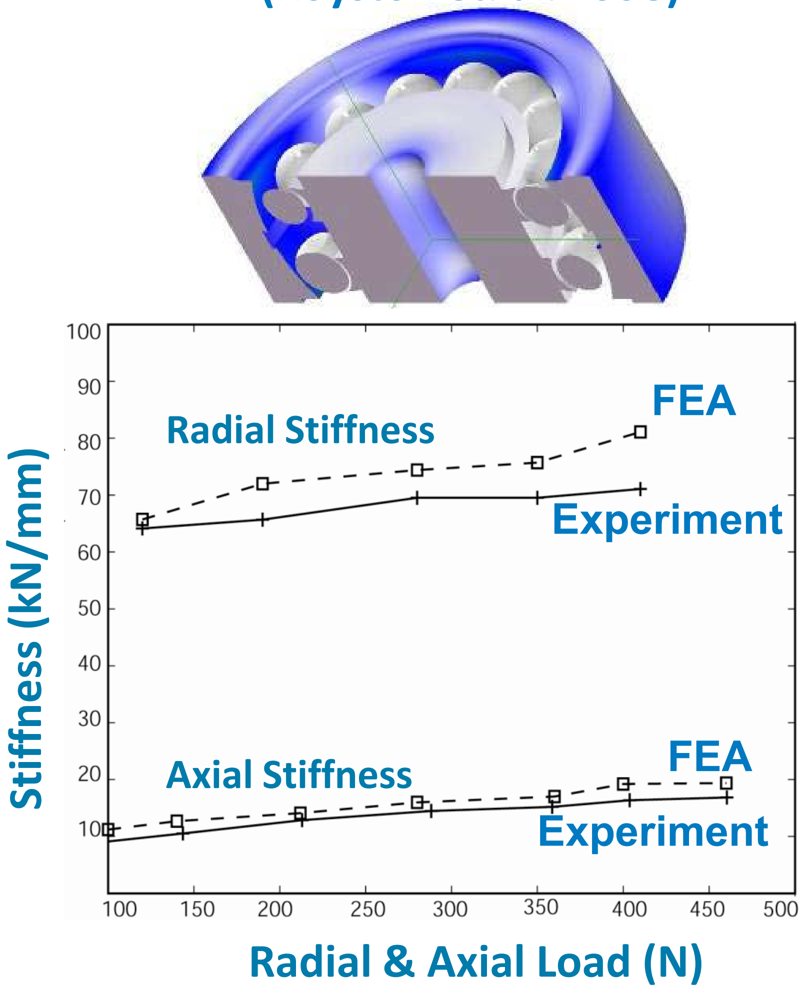
ε : round off 10^{-16}



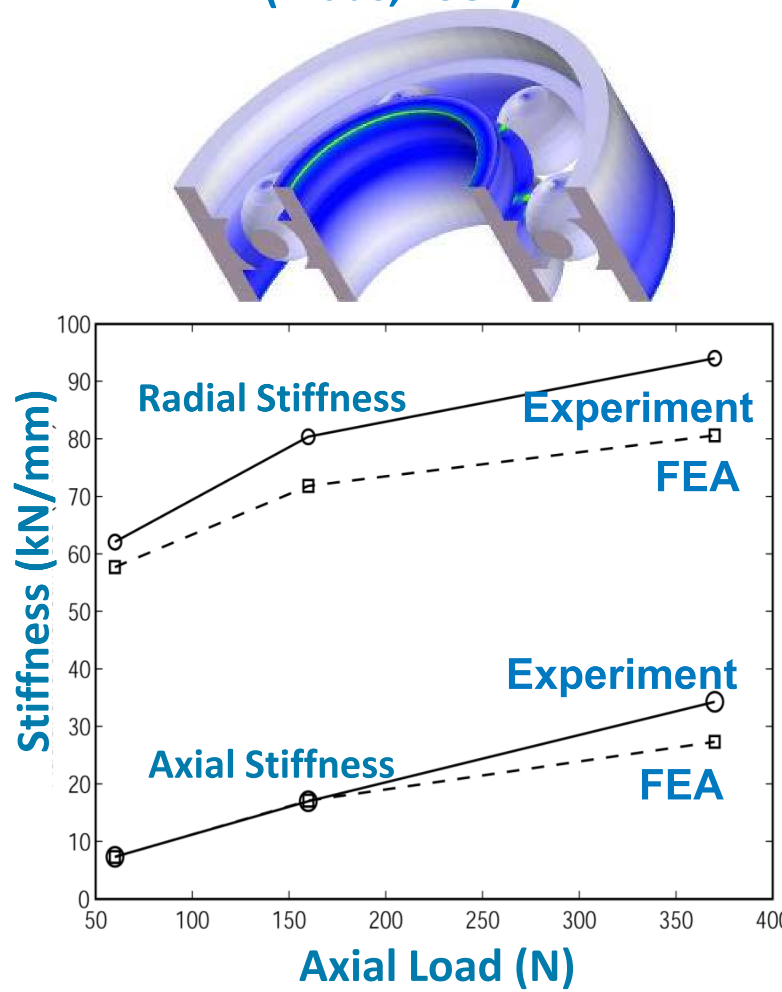
Comparison Against Published Experiments

- **Calculated stiffnesses by FEA agree with experiments**

(Royston et. al. 1998)



(Kraus, 1987)



Comparison Against Advanced Programs

- **Discrepancy is apparent among FEA & advanced models**
- **Differences exist among state-of- the-art tools**

Cylindrical Roller Bearing (FAG N205E)

	FEA	Program A %	Program B%	Program C %
Radial, x (N/mm)	113,149	-16.3%	-66.9%	+10.15%
Radial, y (N/mm)	200,320	+66.0%	-12.9%	+74.63%
Axial (N/mm)	0	0	0	0
Tilting, x (Nmm/rad)	1,843,453	+20.0%	+0.20%	+2.86%
Tilting, y (Nmm/rad)	1,550,232	-59.2%	-73.0%	-57.1%

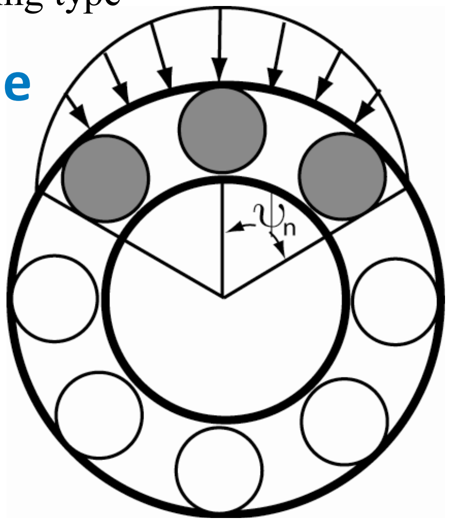
Radial Ball Bearing (SKF Explorer 6205)

	FEA	Program A %	Program B%	Program C %
Radial, x (N/mm)	49,582	-14.5%	+1.40%	-22.73%
Radial, y (N/mm)	95,013	+0.40%	+18.5%	+4.30%
Axial (N/mm)	3,955	-17.1%	-19.6%	-17.8%
Tilting, x (Nmm/rad)	940,694	-4.40%	-1.10%	-3.37%
Tilting, y (Nmm/rad)	506,869	-33.7%	-51.0%	-36.2%

Traditional 2D Bearing Theoretical Model

- **Roller/ball force-deflection relationship**
  - Nonlinear stiffness/Hertzian contact

$$Q_i = k_e \delta_i^n, k_e : \text{stiffness of each roller/ball, where } n \text{ depends on bearing type}$$



- **Sum up all roller reacting forces within contact zone**

$$F = \sum_{\psi_i = 0}^{\psi_i = \pm\psi_n} Q_i \cos\psi_i$$

- **Deflection at specified load F is calculated by**

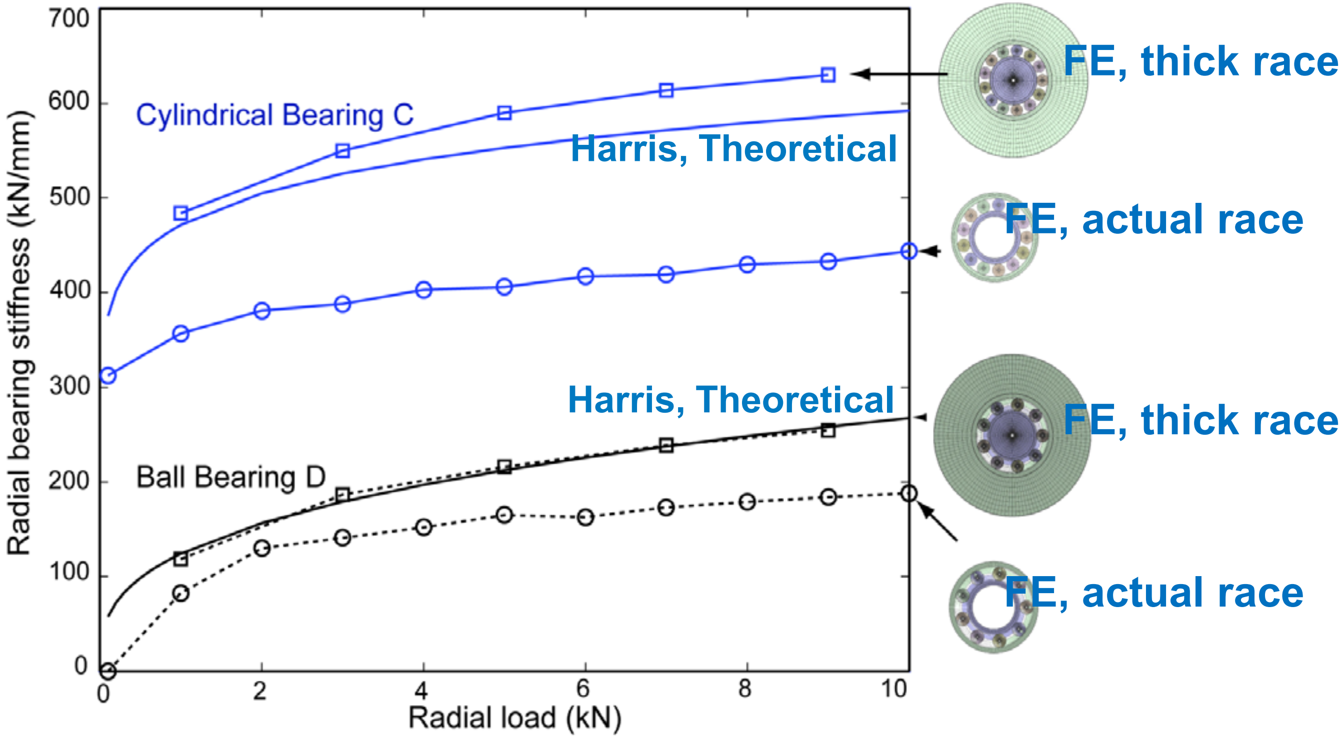
$$F = C\bar{k}_e \delta^n, \bar{k}_e: \text{equivalent stiffness between each roller and races}$$

Major limitations of analytical models

- **Diverse k_e approximations give large discrepancy of stiffness**
- **Assumptions only apply for unrealistic race and roller dimensions**
  - Significantly affects the whole bearing stiffness

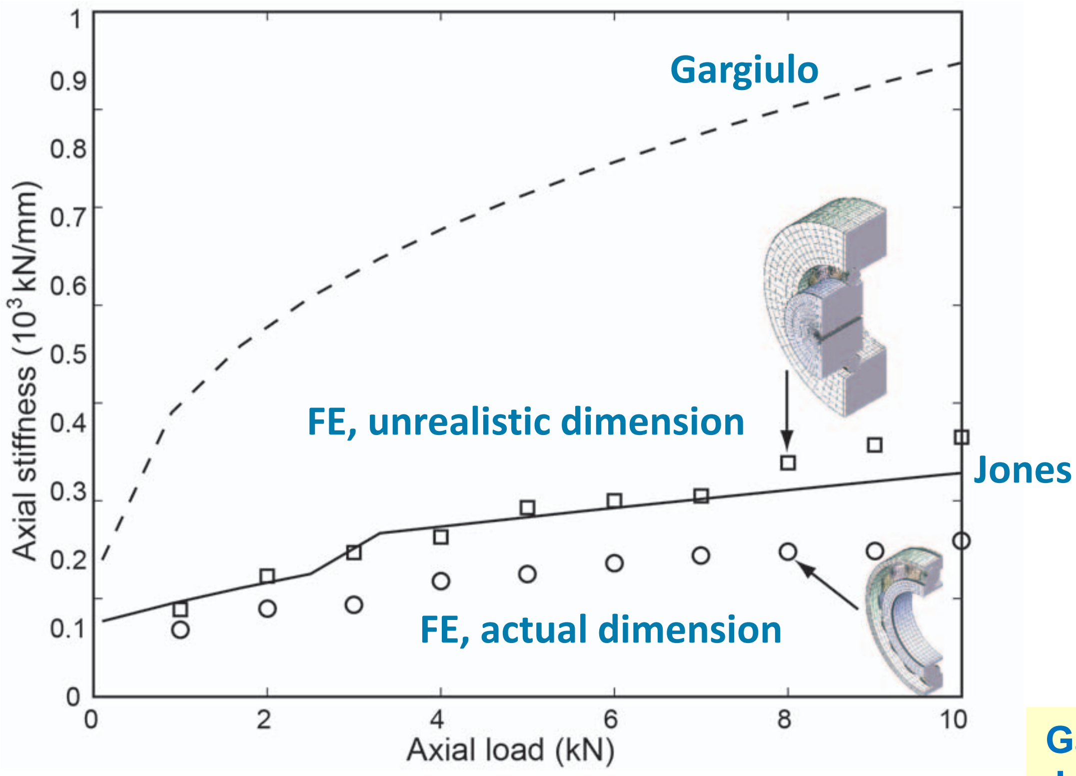
Comparison Against Theoretical Models

- **FEA stiffness agrees with theoretical model**
  - Only with unrealistic races that match Harris's assumptions
- **Theoretical models predict higher stiffness with design dimensions**



Comparison Against Jones's Model

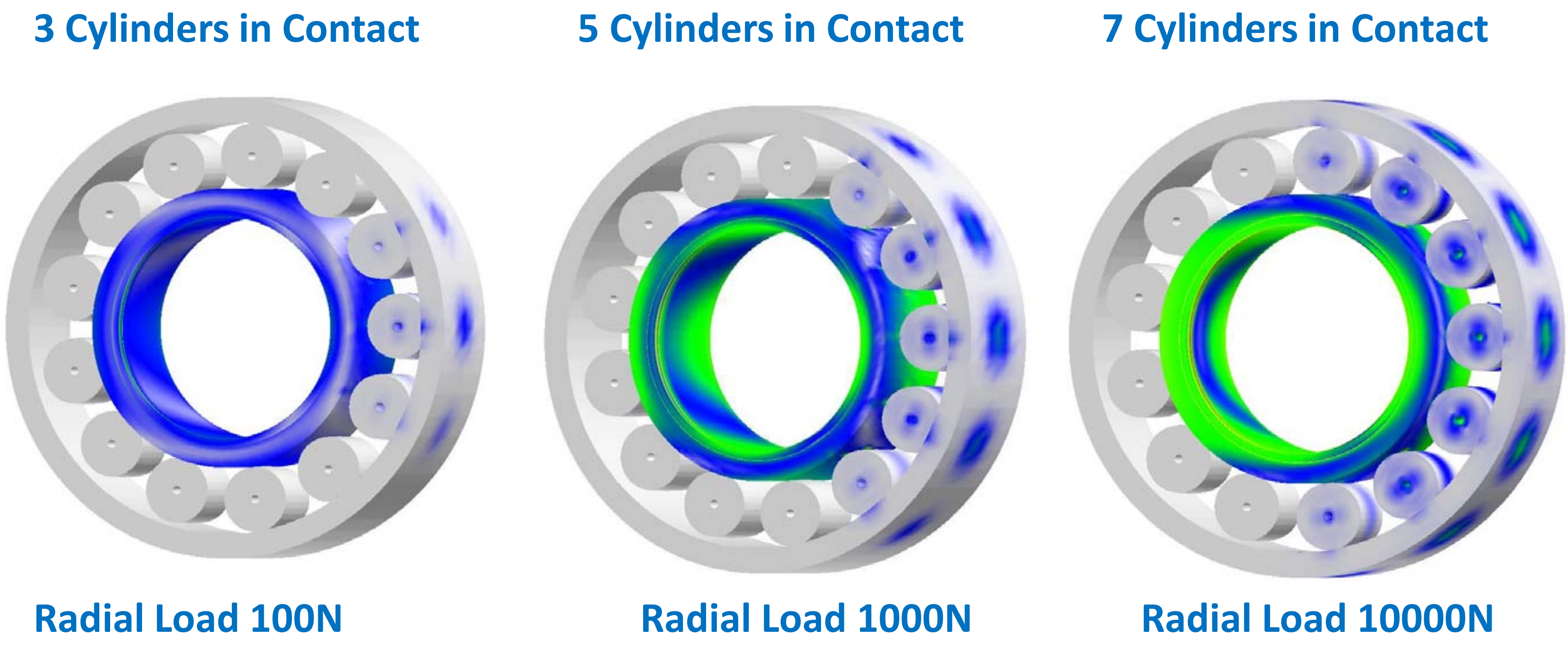
- **Agrees with Jones's model**
  - **Race thickness and length enlarged significantly**
- **Gargiulo's estimate deviates from the other two**



Gargiulo: E.P. Gargiulo(1980)
Jones: A. B. Jones (1946)

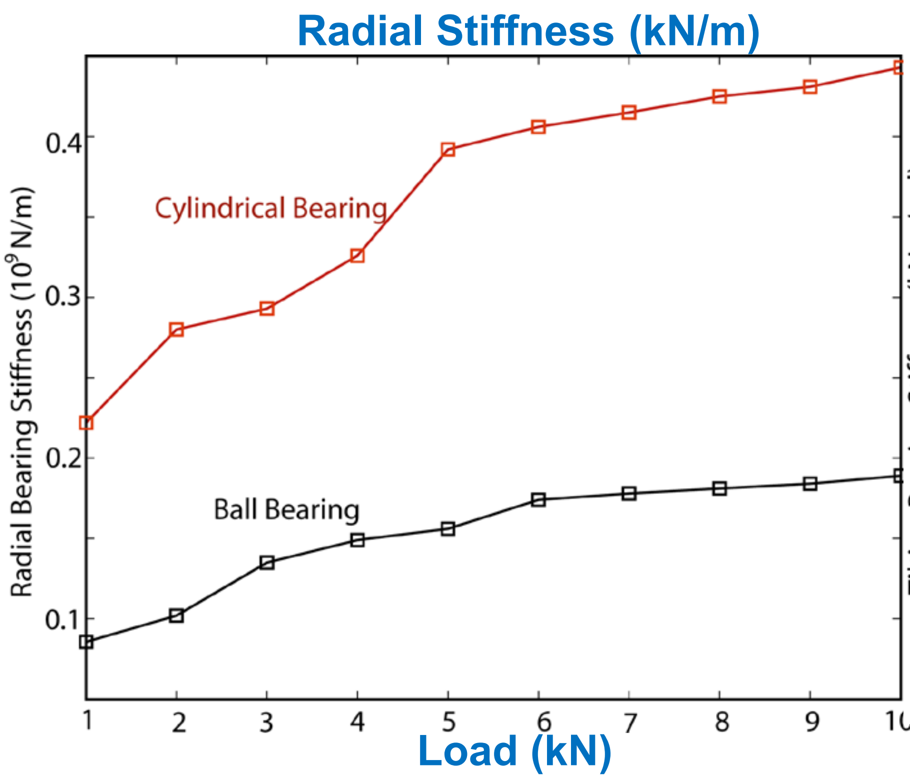
Effect of Applied Load/Torque on Bearings

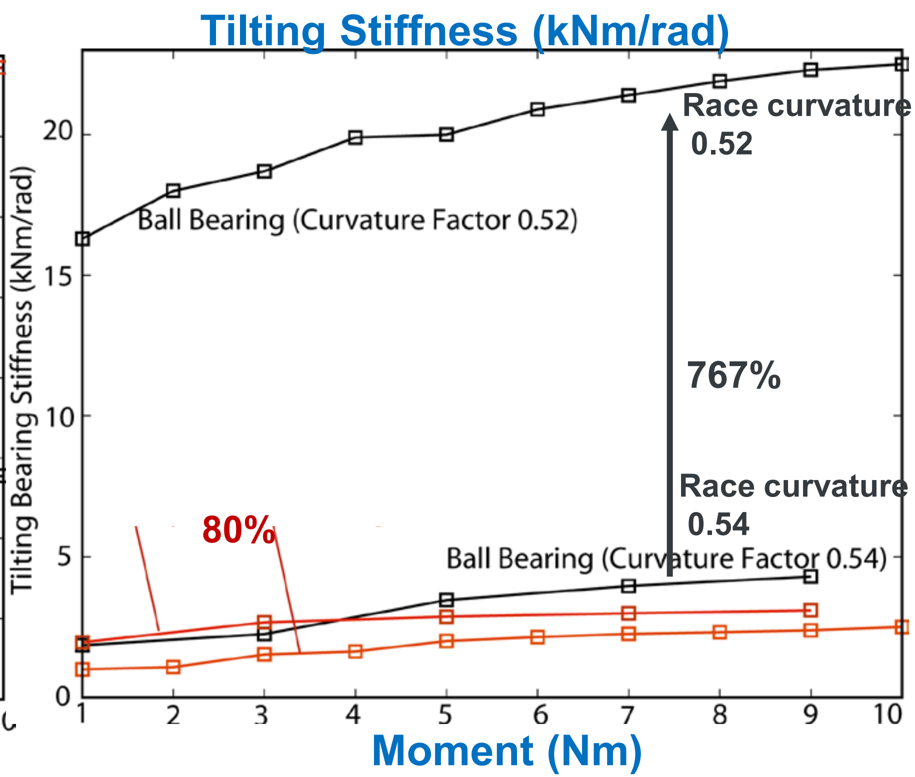
- **Number of rollers in contact changes with torque/load**



Bearing Contact Property

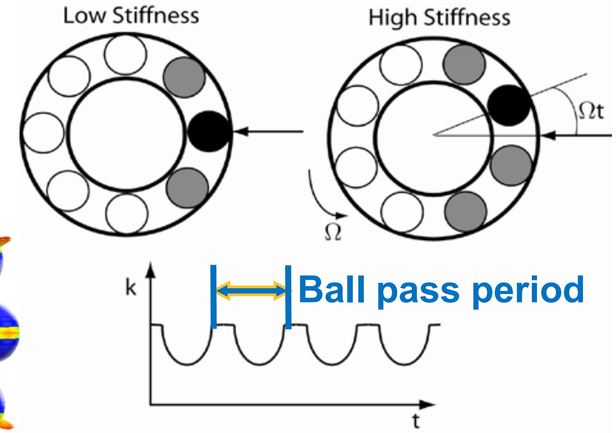
- Stiffness increases nonlinearly with load/torque
- Micro-geometry of bearings highly affect stiffnesses

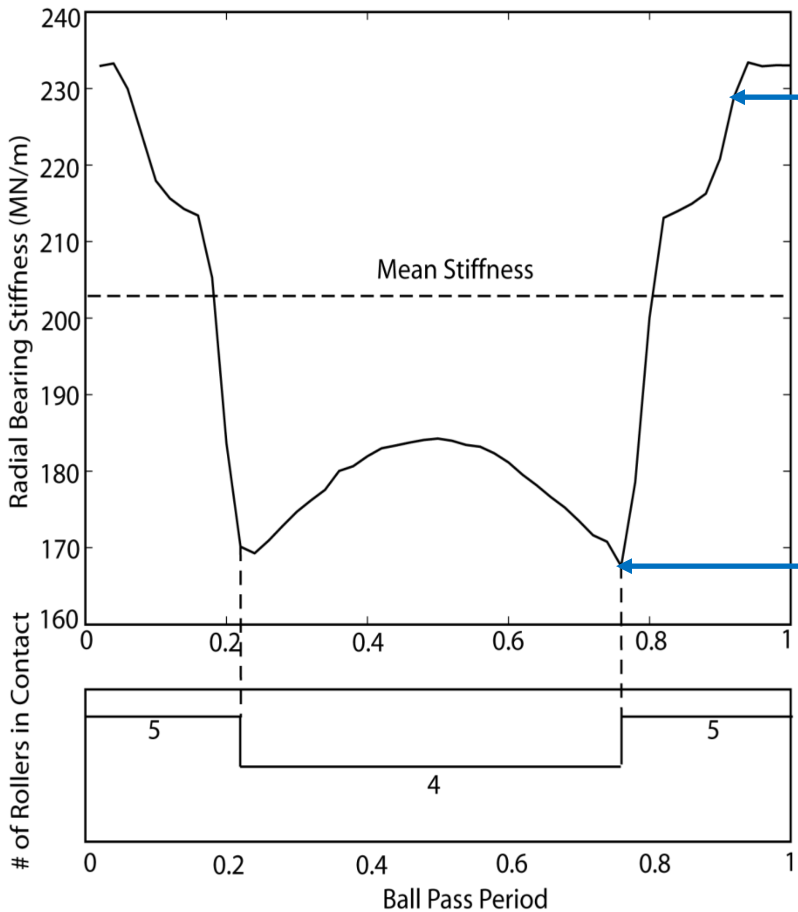


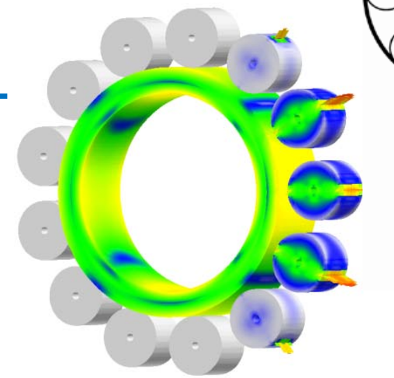


Bearing Stiffness Is Time-Varying: Radial

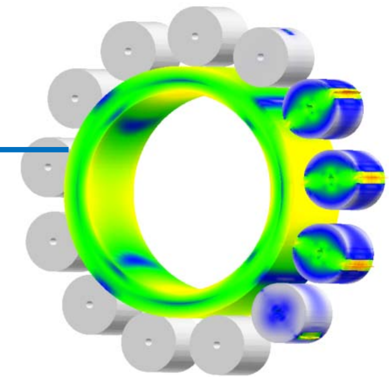
- **Number of rollers in contact changes periodically**
- **Can excite gearbox vibration**







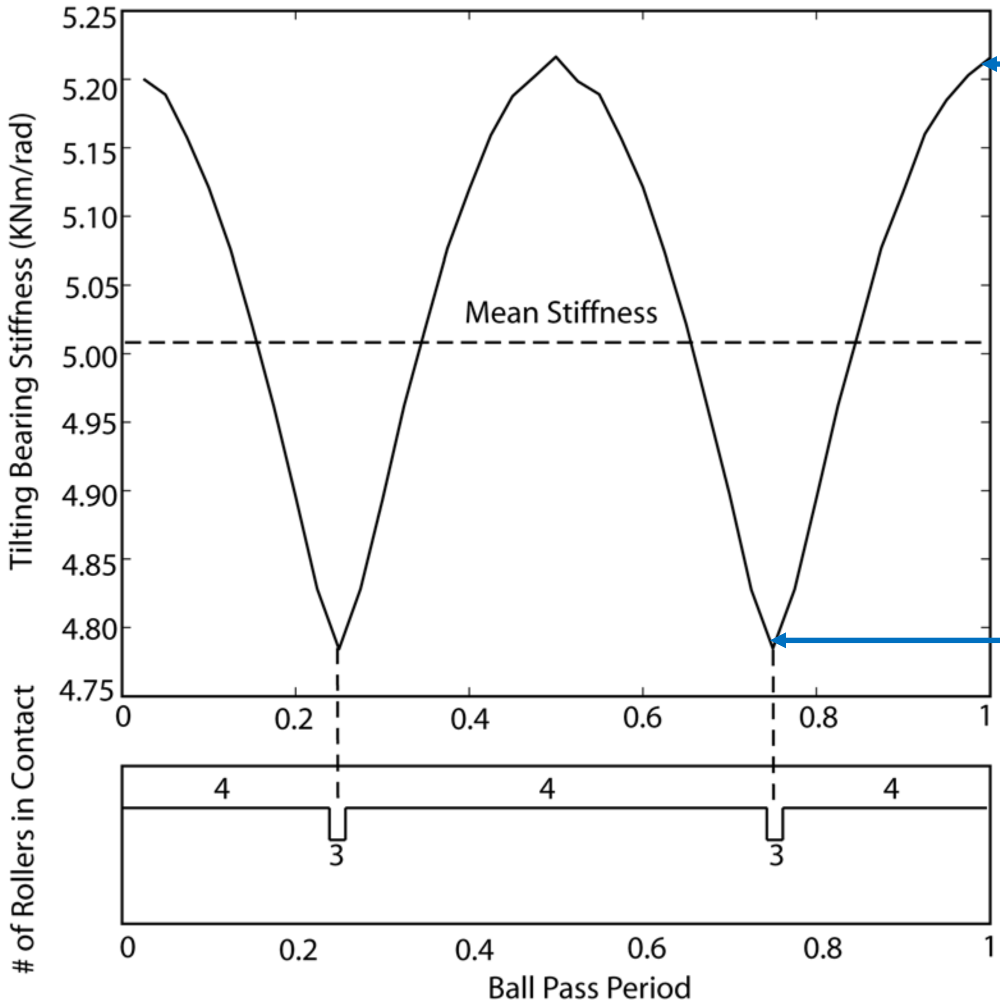
5 Rollers in Contact

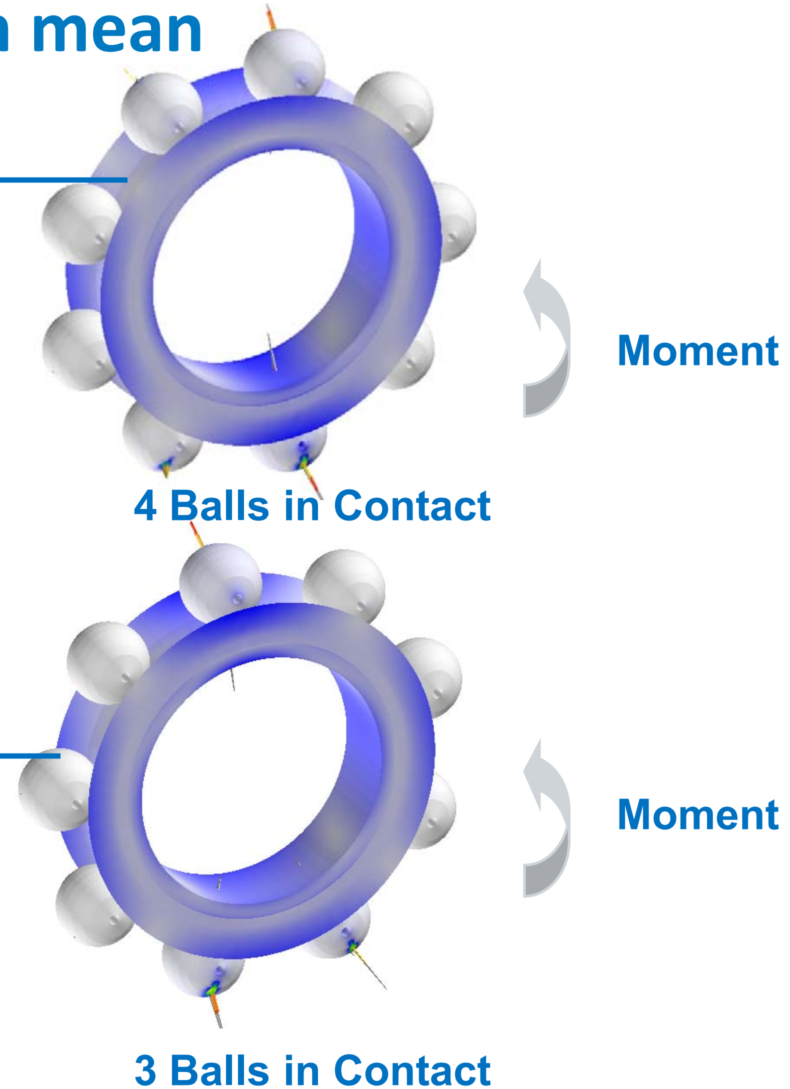


4 Rollers in Contact

Bearing Stiffness Is Time-Varying: Tilting

- **4% maximum deviation from mean**

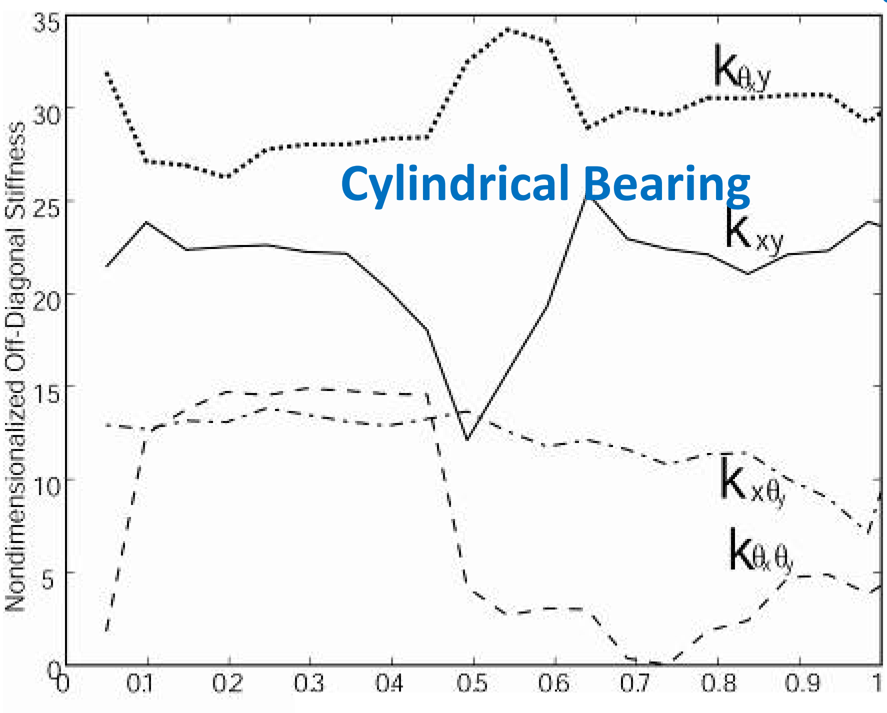




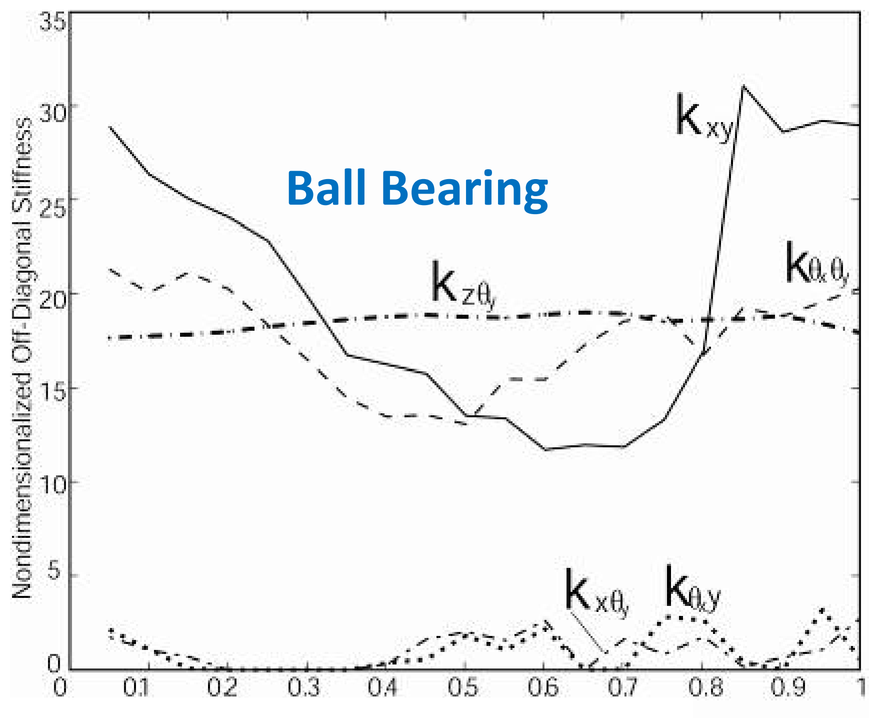
Off-Diagonal Stiffness

- **Stiffness matrix off-diagonal terms are significant**
- **Stiffness fluctuates as bearing rotates**
- **At no instant stiffness matrix is purely diagonal**

Percent of Diagonal Stiffness



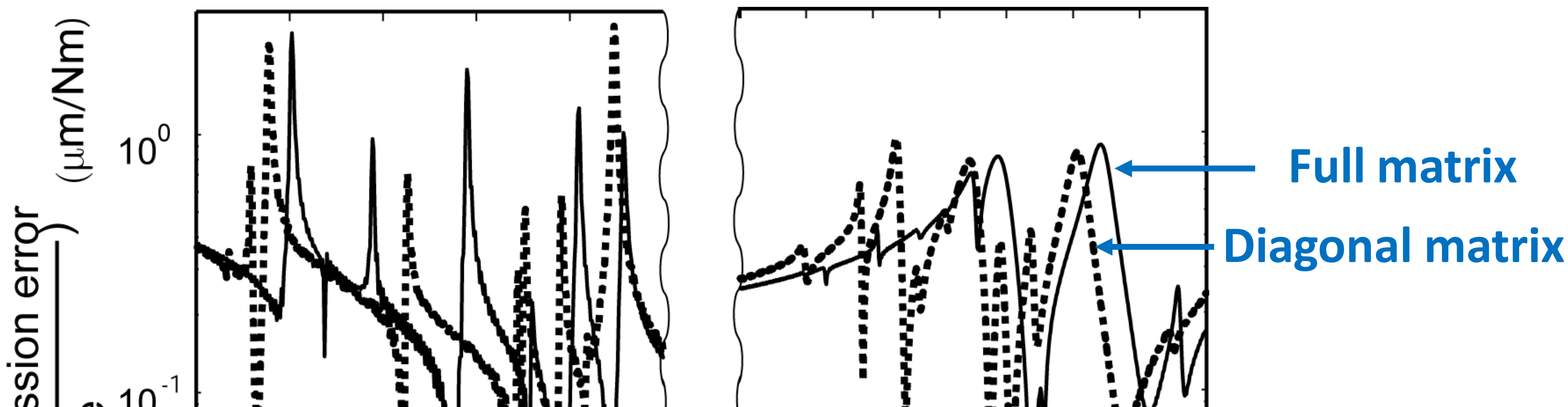
Ball Pass Period

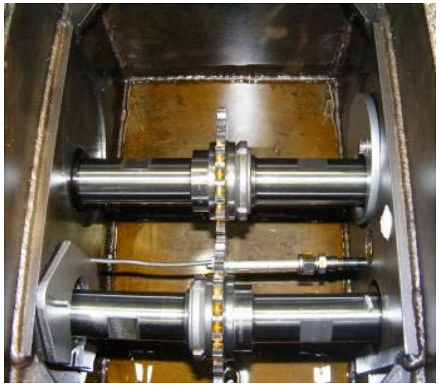


Ball Pass Period

Off-Diagonal Stiffnesses Affect Gear Vibration

- **Gear dynamics with off-diagonal stiffnesses differs from that with diagonal stiffness matrix**
    - Need to include off-diagonal stiffnesses



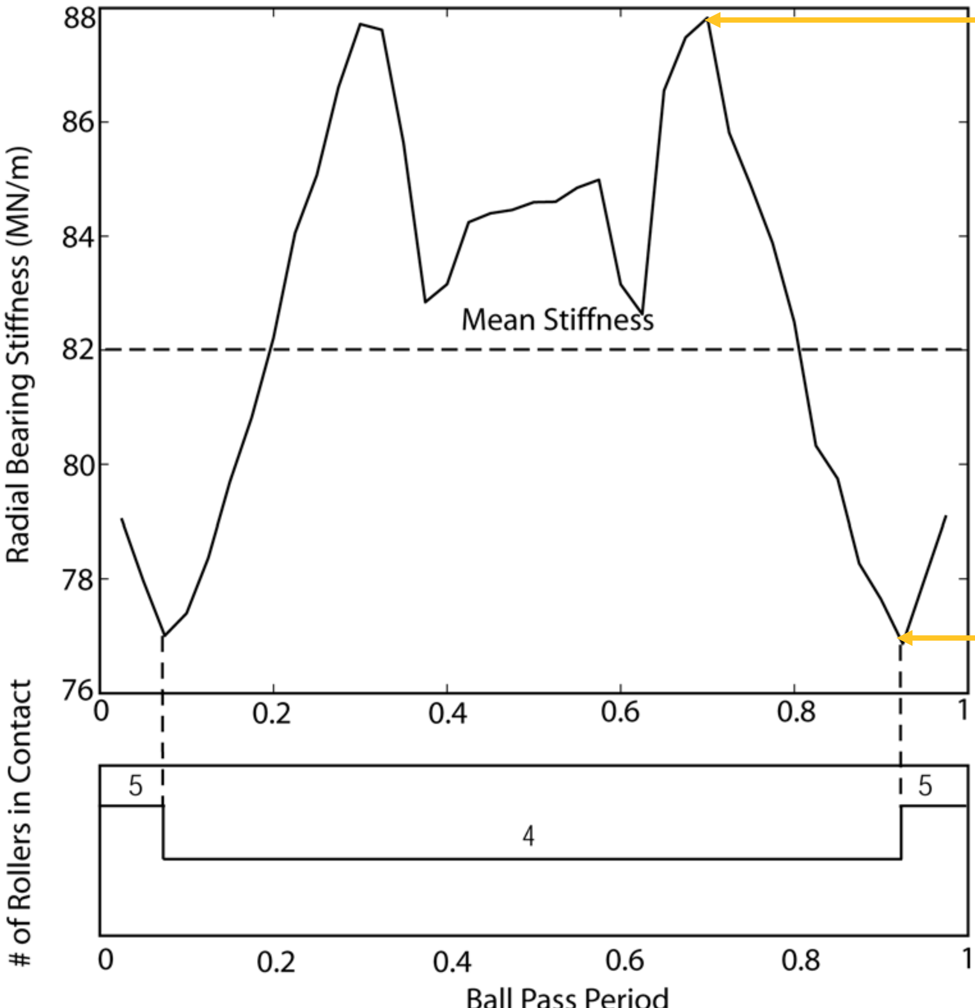


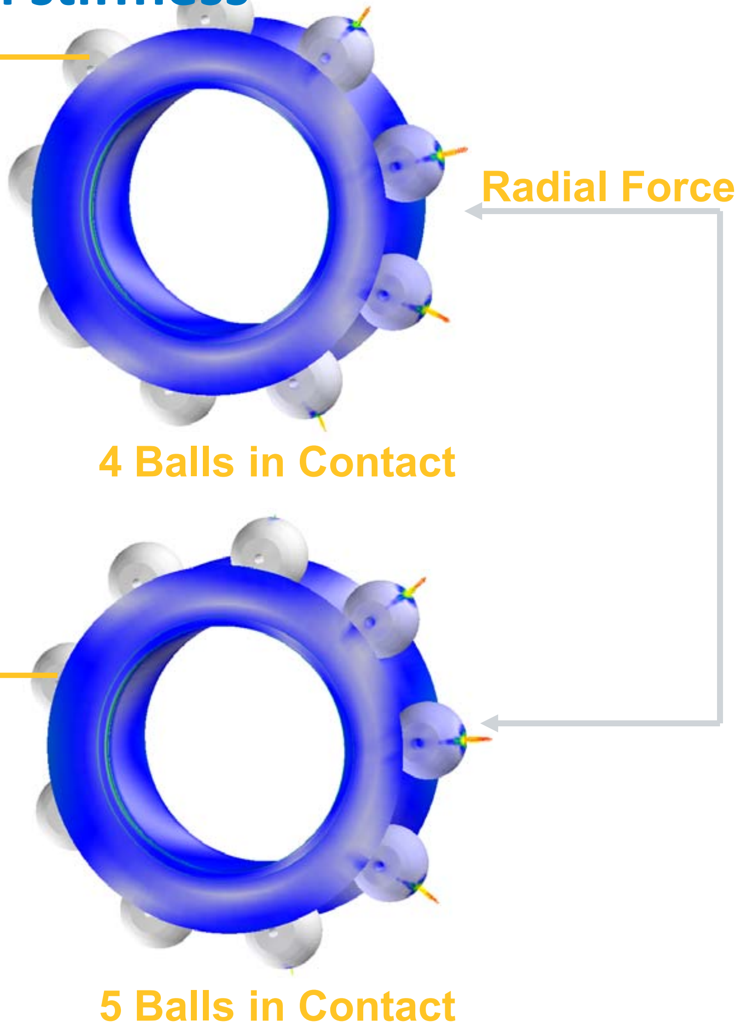
Conclusions

- A method developed to determine fully-populated 6×6 stiffness matrices
- Method validated by experiments
- Comparison against theoretical models expose their limitations
- Bearing contact is nonlinear and time-varying
- Bearing stiffness is sensitive to the microgeometry
- Off-diagonal stiffnesses affect gear dynamics

Radial Stiffness of Ball Bearing

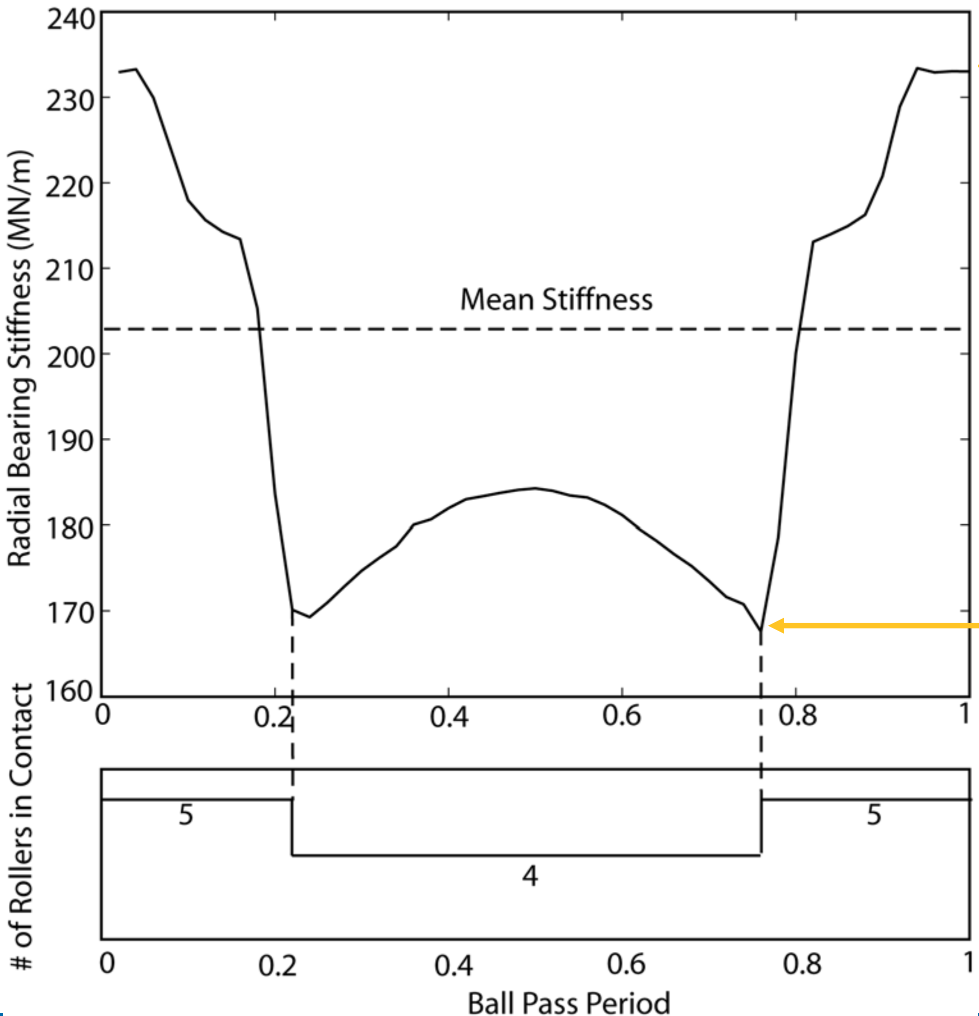
- Number of rollers in contact changes with time
- 7% maximum deviation from mean stiffness

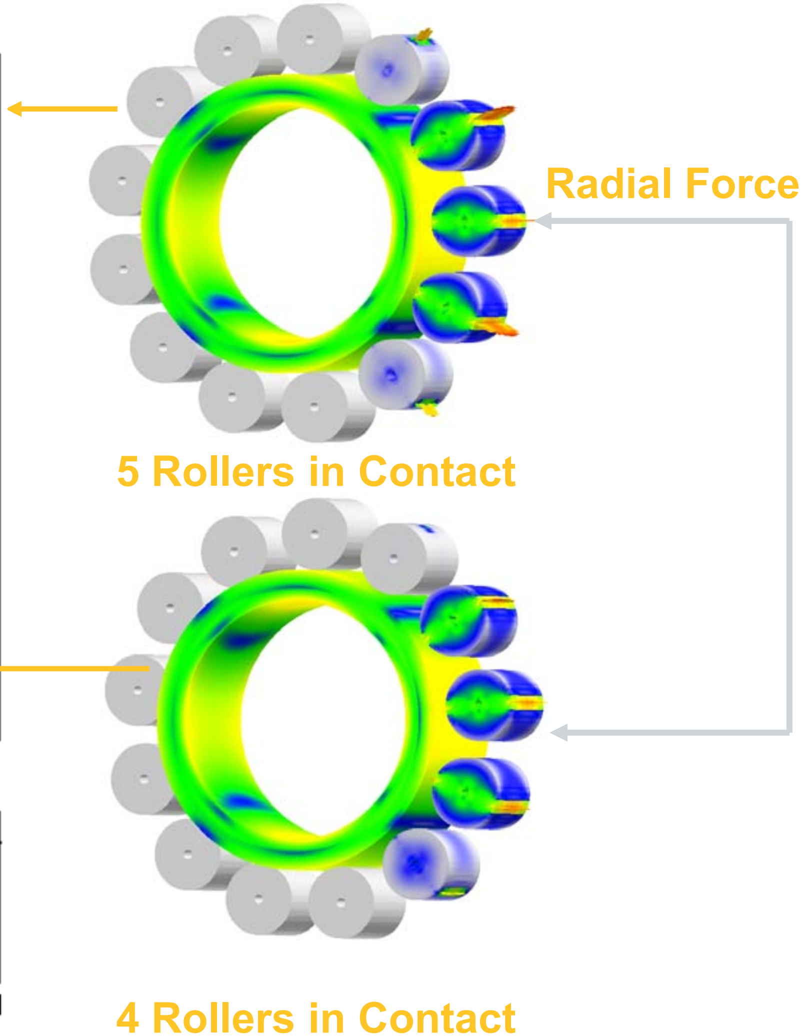




Time-Varying Bearing Stiffness

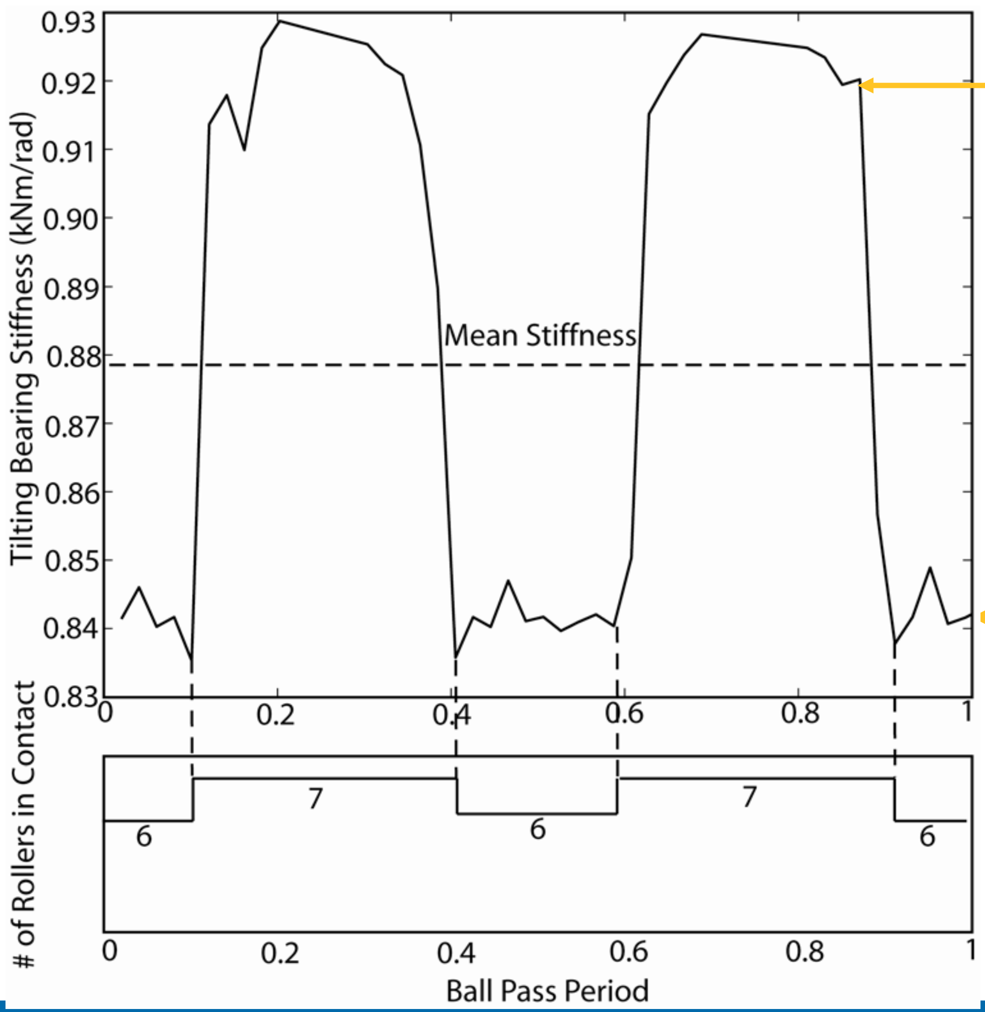
- Number of rollers in contact changes with time
- 16% maximum deviation from mean stiffness

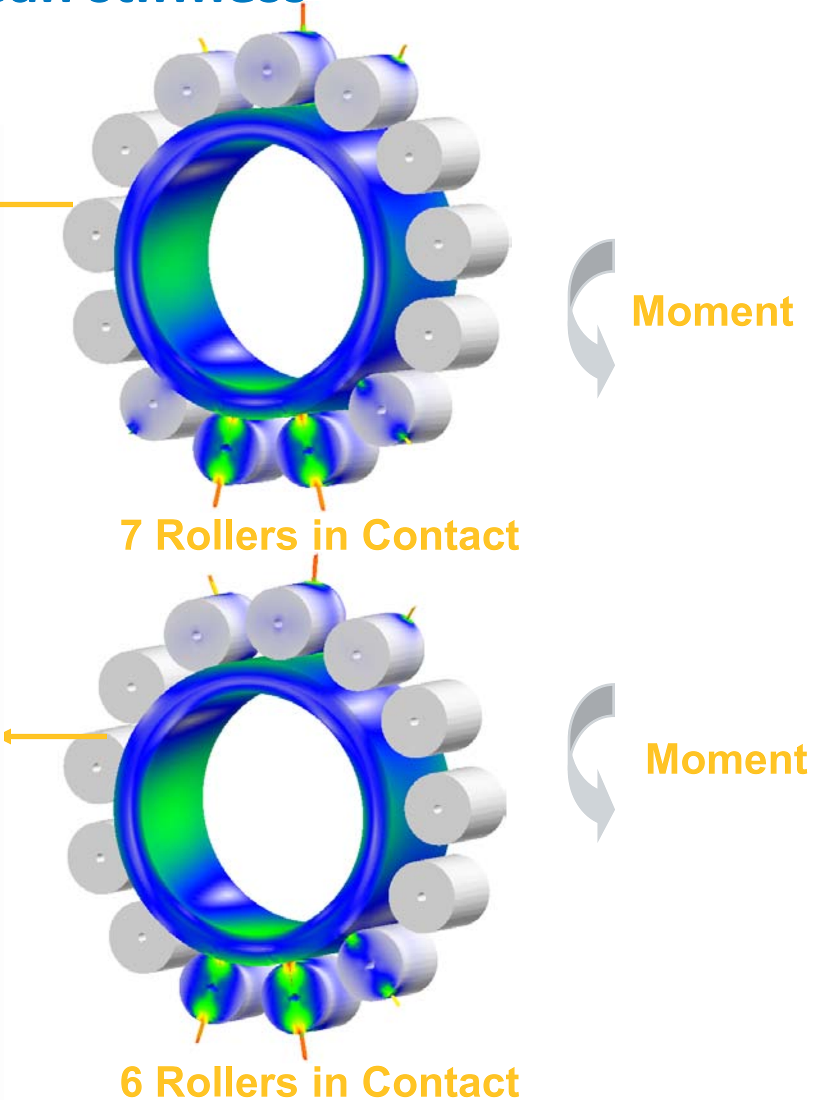




Tilting Stiffness of Cylindrical Bearing

- **6% maximum deviation from mean stiffness**





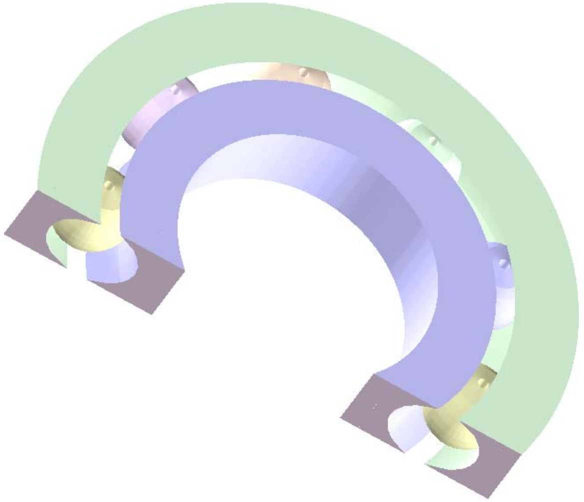
Examples

$$k_{xy} \approx \frac{\left[f_x(X+\delta y) - f_x(X-\delta y)\right]}{2\delta}$$

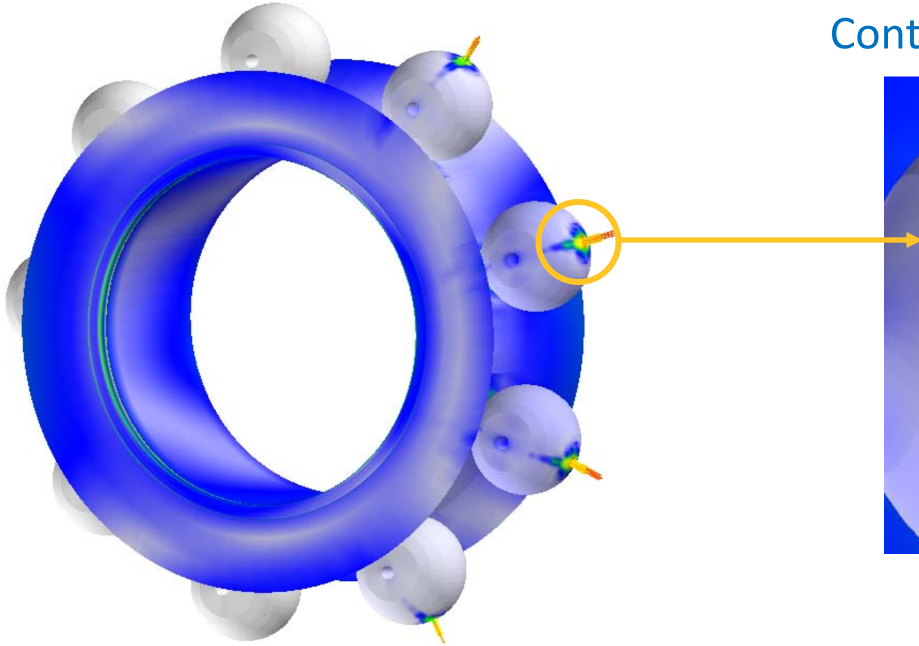
$$k_{\theta_x z} \approx \frac{\left[m_x(X+\delta z) - m_x(X-\delta z)\right]}{2\delta}$$

Deep Groove Ball Bearing

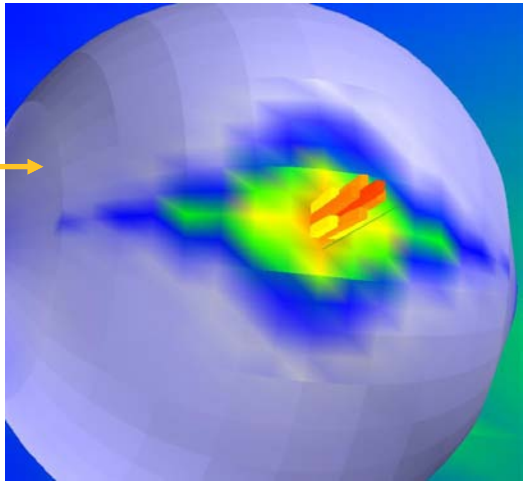
- **Load unevenly distributed on balls**
- **Nominal point contact becomes elliptical contact from elastic deformation**
- **User defines contact grid to accurately capture elliptical contact**



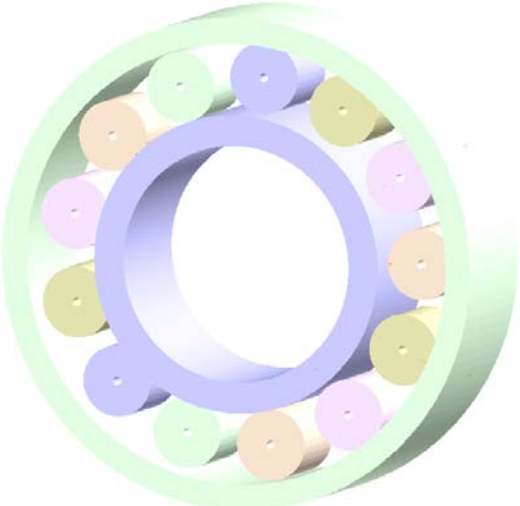
4 Balls in Contact



Contact Pressure under Load

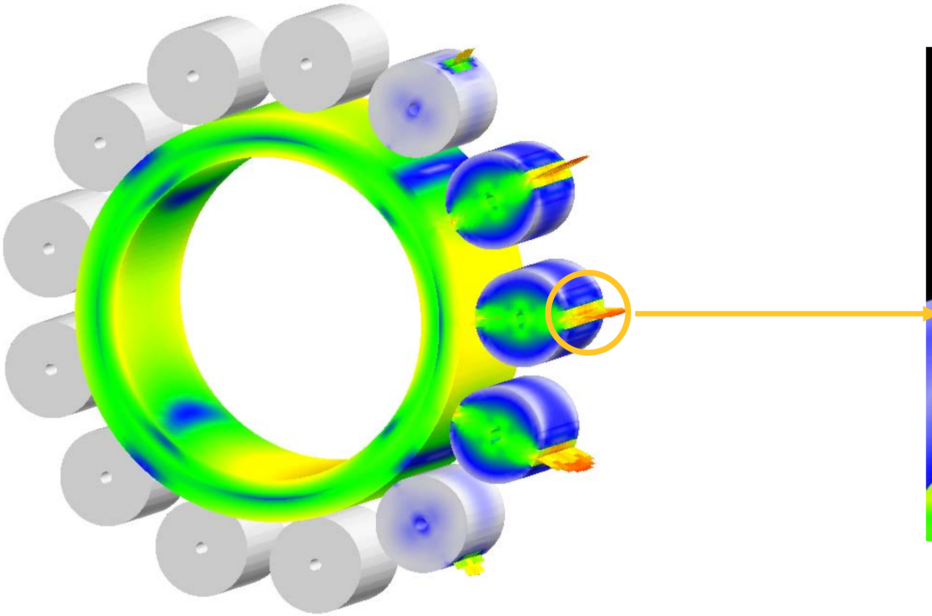


Cylindrical Roller Bearing

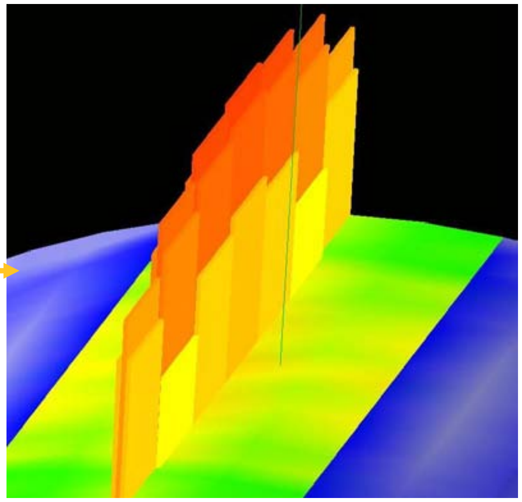


- Load unevenly distributed on rollers
- Nominal line contact becomes square contact from elastic deformation
- User defines contact grid to accurately capture square contact

5 Cylinders in Contact



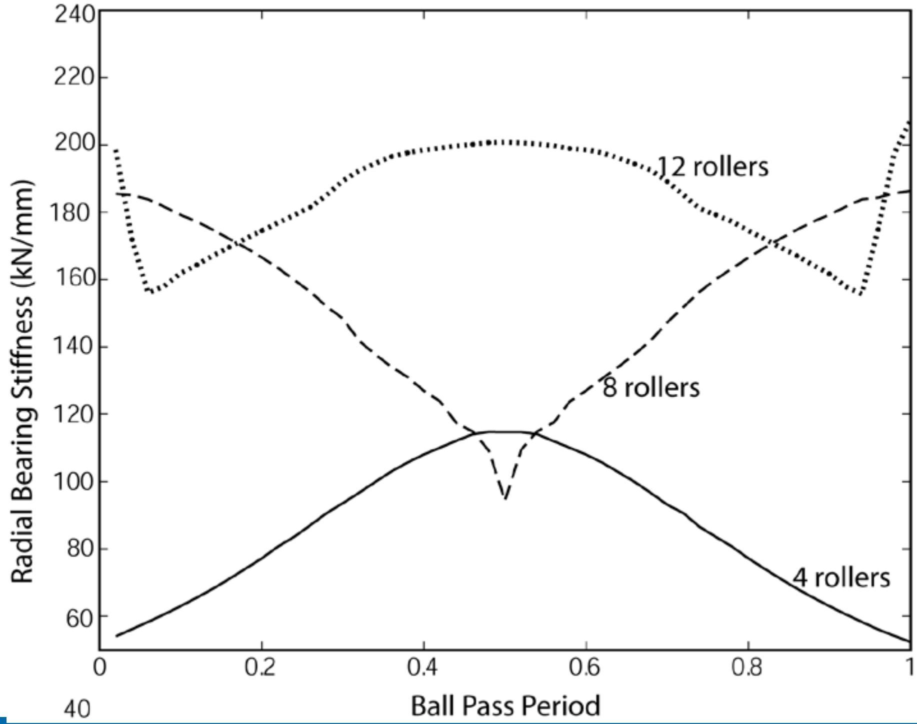
Contact Pressure under Load



Stiffness Changes with Number of Rollers

- Bearing stiffness increases sharply with the number of rollers
- More stiffness variation with odd number of rollers
- Large stiffness fluctuation amplitude with small number of rollers

Even Number of Rollers (4,8,12)



Odd Number of Rollers (5,9,13)